An Algebraic Approach to n-Plithogenic Square Matrices For $18 \leq n \leq 19$

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Abstract

This paper is dedicated to study the algebraic properties and substructures that are related to the symbolic 18 and 19 plithogenic square matrices, where we present many different theorems and examples to clarify the computation of symbolic 18 and 19 plithogenic determinants, and finding 18-plithogenic and 19-plithogenic eigenvalues and eigenvectors. On the other hand, we provide algorithms for computing the inverses of symbolic 18-plithogenic and 19-plithogenic matrices with necessary and sufficient conditions for the orthogonality of those matrices.

Keywords: symbolic 18-plithogenic matrix; symbolic 19-plithogenic matrix; eigenvalue; eigenvector; orthogonal matrix

1. Introduction

The study of plithogenic sets began as a generalization of both fuzzy and neutrosophic sets in [2], where these sets were used in the construction of extended algebraic constructions of classical algebraic structures, where we see a direct use of plithogenic sets in in the expansion of matrices [8,19], vector spaces and modules [3-5], and even integers [6, 17-18].

The study of matrices is fundamental in mathematics in general because of its wide applications, where many researchers around the world have studied them in detail, in terms of finding methods, values and associated eigen-spaces, and also applied in solving equations and ring theory [1, 12-16, 19-25].

Plithogenic matrices of different ranks from the second to the seventeenth rank [7,9, 10-11, 27-28] were studied, where the researchers determined effective algorithms for calculating the determinants of these matrices and their special values, in addition to their orthogonal conditions.

This is what prompted us to follow up the previous efforts and generalize the results to include the plithogenic matrices of the eighteenth and nineteenth ranks, where we were able to find relationships that allow calculating the scales of these matrices and their natural forces, in addition to calculating the special vectors and their special values.

The necessary and sufficient conditions have also been defined for these matrices to be orthogonal.

2. Main Discussion

Definition:

The square 18-plithogenic matrix is defined:

$\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i \,; (\tau_i)_{n \times n}$ is square matrix of real entries.

Example.

Consider the symbolic 18-plithogenic matrix:

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$$\Delta=\begin{pmatrix}-6 & -9\\ 1 & 56\end{pmatrix}+\begin{pmatrix}1 & 12\\ 65 & 31\end{pmatrix}P_1+\begin{pmatrix}123 & -1\\ 12 & 31\end{pmatrix}P_2+\begin{pmatrix}-96 & -8\\ -19 & -16\end{pmatrix}P_3+\begin{pmatrix}38 & 65\\ 116 & 31\end{pmatrix}P_4+\begin{pmatrix}-5 & -5\\ -5 & -2\end{pmatrix}P_5+\begin{pmatrix}3 & -1\\ 3 & -2\end{pmatrix}P_6+\begin{pmatrix}-1 & 7\\ 9 & 8\end{pmatrix}P_7+\begin{pmatrix}12 & 11\\ 65 & -1\end{pmatrix}P_8+\begin{pmatrix}-1 & 19\\ -1 & 0\end{pmatrix}P_{10}+\begin{pmatrix}8 & -1\\ 7 & 5\end{pmatrix}P_{11}+\begin{pmatrix}42 & -41\\ 2 & -8\end{pmatrix}P_{12}+\begin{pmatrix}41 & -1\\ 21 & -9\end{pmatrix}P_{13}+\begin{pmatrix}4 & -11\\ 21 & -8\end{pmatrix}P_{14}+\begin{pmatrix}41 & -1\\ 2 & -8\end{pmatrix}P_{15}+\begin{pmatrix}41 & -31\\ 32 & -8\end{pmatrix}P_{16}+\begin{pmatrix}41 & -1\\ 2 & -8\end{pmatrix}P_{17}+\begin{pmatrix}-1 & 19\\ -1 & 0\end{pmatrix}P_{18}.$$

Definition.

Let $\tau=\tau_0+\sum_{i=1}^{18}\tau_iP_i$ be a symbolic 18-plithogenic matrix of size $n\times n$, hence:

$\det\tau=\det(\tau_0)+[\det(\sum_{i=0}^{1}\tau_i)-\det(\tau_0)]P_1+[\det(\sum_{i=0}^{2}\tau_i)-\det(\sum_{i=0}^{1}\tau_i)]P_2+[\det(\sum_{i=0}^{3}\tau_i)-\det(\sum_{i=0}^{2}\tau_i)]P_3+[\det(\sum_{i=0}^{4}\tau_i)-\det(\sum_{i=0}^{3}\tau_i)]P_4+[\det(\sum_{i=0}^{5}\tau_i)-\det(\sum_{i=0}^{4}\tau_i)]P_5+[\det(\sum_{i=0}^{6}\tau_i)-\det(\sum_{i=0}^{5}\tau_i)]P_6+[\det(\sum_{i=0}^{7}\tau_i)-\det(\sum_{i=0}^{6}\tau_i)]P_7+[\det(\sum_{i=0}^{8}\tau_i)-\det(\sum_{i=0}^{7}\tau_i)]P_8+[\det(\sum_{i=0}^{9}\tau_i)-\det(\sum_{i=0}^{8}\tau_i)]P_9+[\det(\sum_{i=0}^{10}\tau_i)-\det(\sum_{i=0}^{9}\tau_i)]P_{10}+[\det(\sum_{i=0}^{11}\tau_i)-\det(\sum_{i=0}^{10}\tau_i)]P_{11}+[\det(\sum_{i=0}^{12}\tau_i)-\det(\sum_{i=0}^{11}\tau_i)]P_{12}+[\det(\sum_{i=0}^{13}\tau_i)-\det(\sum_{i=0}^{12}\tau_i)]P_{13}+[\det(\sum_{i=0}^{14}\tau_i)-\det(\sum_{i=0}^{13}\tau_i)]P_{14}+[\det(\sum_{i=0}^{15}\tau_i)-\det(\sum_{i=0}^{14}\tau_i)]P_{15}+[\det(\sum_{i=0}^{16}\tau_i)-\det(\sum_{i=0}^{15}\tau_i)]P_{16}+[\det(\sum_{i=0}^{17}\tau_i)-\det(\sum_{i=0}^{16}\tau_i)]P_{17}+[\det(\sum_{i=0}^{18}\tau_i)-\det(\sum_{i=0}^{17}\tau_i)]P_{18}.$

Theorem

Let $\tau=\tau_0+\sum_{i=1}^{18}\tau_iP_i$ be a symbolic 18-plithogenic matrix of size $n\times n$, hence:

1. τ is invertible if and only if $\det\tau$ is an invertible symbolic 18-plithogenic real number.
2. $\tau^{-1}={\tau_0}^{-1}+[(\sum_{i=0}^{1}\tau_i)^{-1}-{\tau_0}^{-1}]P_1+[(\sum_{i=0}^{2}\tau_i)^{-1}-(\sum_{i=0}^{1}\tau_i)^{-1}]P_2+[(\sum_{i=1}^{3}\tau_i)^{-1}-(\sum_{i=0}^{2}\tau_i)^{-1}]P_3+[(\sum_{i=1}^{4}\tau_i)^{-1}-(\sum_{i=0}^{3}\tau_i)^{-1}]P_4+[(\sum_{i=1}^{5}\tau_i)^{-1}-(\sum_{i=0}^{4}\tau_i)^{-1}]P_5+[(\sum_{i=1}^{6}\tau_i)^{-1}-(\sum_{i=0}^{5}\tau_i)^{-1}]P_6+[(\sum_{i=1}^{7}\tau_i)^{-1}-(\sum_{i=0}^{6}\tau_i)^{-1}]P_7+[(\sum_{i=1}^{8}\tau_i)^{-1}-(\sum_{i=0}^{7}\tau_i)^{-1}]P_8+[(\sum_{i=1}^{9}\tau_i)^{-1}-(\sum_{i=0}^{8}\tau_i)^{-1}]P_9+[(\sum_{i=1}^{10}\tau_i)^{-1}-(\sum_{i=0}^{9}\tau_i)^{-1}]P_{10}+[(\sum_{i=1}^{11}\tau_i)^{-1}-(\sum_{i=0}^{10}\tau_i)^{-1}]P_{11}+[(\sum_{i=1}^{12}\tau_i)^{-1}-(\sum_{i=0}^{11}\tau_i)^{-1}]P_{12}+[(\sum_{i=1}^{13}\tau_i)^{-1}-(\sum_{i=0}^{12}\tau_i)^{-1}]P_{13}+[(\sum_{i=1}^{14}\tau_i)^{-1}-(\sum_{i=0}^{13}\tau_i)^{-1}]P_{14}+[(\sum_{i=1}^{15}\tau_i)^{-1}-(\sum_{i=0}^{14}\tau_i)^{-1}]P_{15}+[(\sum_{i=1}^{16}\tau_i)^{-1}-(\sum_{i=0}^{15}\tau_i)^{-1}]P_{16}+[(\sum_{i=1}^{17}\tau_i)^{-1}-(\sum_{i=0}^{16}\tau_i)^{-1}]P_{17}+[(\sum_{i=1}^{18}\tau_i)^{-1}-(\sum_{i=0}^{17}\tau_i)^{-1}]P_{18}.$

Proof

1). Let $\tau=\tau_0+\sum_{i=1}^{18}\tau_iP_i$, then τ is invertible if and only if there exists $I=I_0+\sum_{i=1}^{18}I_iP_i$ such that:
$\tau\times I=U_{n\times n}$, hence:

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$$\begin{cases}
\tau_0 I_0 = U_{n\times n} \\
\sum_{i=0}^{1} \tau_i \sum_{i=0}^{1} I_i - \tau_0 I_0 = O_{n\times n} \\
\sum_{i=0}^{2} \tau_i \sum_{i=0}^{2} I_i - \sum_{i=0}^{1} \tau_i \sum_{i=0}^{1} I_i = O_{n\times n} \\
\sum_{i=0}^{3} \tau_i \sum_{i=0}^{3} I_i - \sum_{i=0}^{2} \tau_i \sum_{i=0}^{2} I_i = O_{n\times n} \\
\sum_{i=0}^{4} \tau_i \sum_{i=0}^{4} I_i - \sum_{i=0}^{3} \tau_i \sum_{i=0}^{3} I_i = O_{n\times n} \\
\sum_{i=0}^{5} \tau_i \sum_{i=0}^{5} I_i - \sum_{i=0}^{4} \tau_i \sum_{i=0}^{4} I_i = O_{n\times n} \\
\sum_{i=0}^{6} \tau_i \sum_{i=0}^{6} I_i - \sum_{i=0}^{5} \tau_i \sum_{i=0}^{5} I_i = O_{n\times n} \\
\sum_{i=0}^{7} \tau_i \sum_{i=0}^{7} I_i - \sum_{i=0}^{6} \tau_i \sum_{i=0}^{6} I_i = O_{n\times n} \\
\sum_{i=0}^{8} \tau_i \sum_{i=0}^{8} I_i - \sum_{i=0}^{7} \tau_i \sum_{i=0}^{7} I_i = O_{n\times n} \\
\sum_{i=0}^{9} \tau_i \sum_{i=0}^{9} I_i - \sum_{i=0}^{8} \tau_i \sum_{i=0}^{8} I_i = O_{n\times n} \\
\sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} I_i - \sum_{i=0}^{9} \tau_i \sum_{i=0}^{9} I_i = O_{n\times n} \\
\sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} I_i - \sum_{i=0}^{10} \tau_i \sum_{i=0}^{10} I_i = O_{n\times n} \\
\sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} I_i - \sum_{i=0}^{11} \tau_i \sum_{i=0}^{11} I_i = O_{n\times n} \\
\sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} I_i - \sum_{i=0}^{12} \tau_i \sum_{i=0}^{12} I_i = O_{n\times n} \\
\sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} I_i - \sum_{i=0}^{13} \tau_i \sum_{i=0}^{13} I_i = O_{n\times n} \\
\sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} I_i - \sum_{i=0}^{14} \tau_i \sum_{i=0}^{14} I_i = O_{n\times n} \\
\sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} I_i - \sum_{i=0}^{15} \tau_i \sum_{i=0}^{15} I_i = O_{n\times n} \\
\sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} I_i - \sum_{i=0}^{16} \tau_i \sum_{i=0}^{16} I_i = O_{n\times n} \\
\sum_{i=0}^{18} \tau_i \sum_{i=0}^{18} I_i - \sum_{i=0}^{17} \tau_i \sum_{i=0}^{17} I_i = O_{n\times n}
\end{cases}$$

This implies that:

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$$\begin{cases} \tau_0 I_0 = U_{n\times n} \\ \sum_{i=0}^{r} \tau_i \sum_{i=0}^{r} I_i = U_{n\times n} \end{cases}; \ 1 \le r \le 18$$

Hence $\det(\sum_{i=0}^{r} \tau_i) \neq 0$ for all $1 \le r \le 18$, so that $\det(\tau)$ is invertible in $18 - SP_R$.

2). $\sum_{i=0}^{r} I_i = (\sum_{i=0}^{r} \tau_i)^{-1}$for $1 \le r \le 18$, therefor

$$\tau^{-1} = {\tau_0}^{-1} + [(\sum_{i=0}^{1} \tau_i)^{-1} - {\tau_0}^{-1}]P_1 + [(\sum_{i=0}^{2} \tau_i)^{-1} - (\sum_{i=0}^{1} \tau_i)^{-1}]P_2 + [(\sum_{i=0}^{3} \tau_i)^{-1} - (\sum_{i=0}^{2} \tau_i)^{-1}]P_3 + [(\sum_{i=0}^{4} \tau_i)^{-1} - (\sum_{i=0}^{3} \tau_i)^{-1}]P_4 + \left[\left(\sum_{i=0}^{5} \tau_i\right)^{-1} - (\sum_{i=0}^{4} \tau_i)^{-1}\right] P_5 + \left[(\sum_{i=0}^{6} \tau_i)^{-1} - \left(\sum_{i=0}^{5} \tau_i\right)^{-1}\right] P_6 + [(\sum_{i=0}^{7} \tau_i)^{-1} - (\sum_{i=0}^{6} \tau_i)^{-1}]P_7 + [(\sum_{i=0}^{8} \tau_i)^{-1} - (\sum_{i=0}^{7} \tau_i)^{-1}]P_8 + [(\sum_{i=0}^{9} \tau_i)^{-1} - (\sum_{i=0}^{8} \tau_i)^{-1}]P_9 + [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^{9} \tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + \left[\left(\sum_{i=0}^{15} \tau_i\right)^{-1} - (\sum_{i=0}^{14} \tau_i)^{-1}\right] P_{15} + \left[(\sum_{i=0}^{16} \tau_i)^{-1} - \left(\sum_{i=0}^{15} \tau_i\right)^{-1}\right] P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - (\sum_{i=0}^{17} \tau_i)^{-1}]P_{18}.$$

Definition.

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{18} \varepsilon_i P_i$ be a symbolic 18-plithogenic real number and $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ be a symbolic 18-plithogenic square real matrix, then ε is called symbolic 18-plithogenic eigen value if and only if $\tau X = \varepsilon X$.

X is called symbolic 18-plithogenic eigenvector.

Theorem:

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{18} \varepsilon_i P_i \in 18 - SP_R$, $X = X_0 + \sum_{i=1}^{18} X_i P_i$ be a symbolic 18-plithogenic vector, then ε is eigen value of $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ with X as the corresponding eigen vector if and only if:

$\sum_{i=0}^{j} \varepsilon_i$ is eigen value of $\sum_{i=0}^{j} \tau_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \le j \le 18$.

Proof:

It is clear that r is an eigen value of τ with X as an eigen vector if and only if:

$\tau . X = \varepsilon . X$, which is equivalent to:

$$\begin{cases} \tau_0 X_0 = \varepsilon_0 X_0 \\ \sum_{i=0}^{j} \tau_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} \varepsilon_i \sum_{i=0}^{j} X_i \end{cases}; \ 1 \le j \le 18$$

Which is equivalent to:

$\sum_{i=0}^{j} \varepsilon_i$ is an eigen value of $\sum_{i=0}^{j} \tau_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 18$.

Theorem:

$$\begin{aligned} \tau^n = {\tau_0}^n + P_1 \left[\left(\sum_{i=0}^{1} \tau_i\right)^n - {\tau_0}^n\right] + \left[\left(\sum_{i=0}^{2} \tau_i\right)^n - \left(\sum_{i=0}^{1} \tau_i\right)^n\right] P_2 + \left[\left(\sum_{i=0}^{3} \tau_i\right)^n - \left(\sum_{i=0}^{2} \tau_i\right)^n\right] P_3 \\ + \left[\left(\sum_{i=0}^{4} \tau_i\right)^n - \left(\sum_{i=0}^{3} \tau_i\right)^n\right] P_4 + \left[\left(\sum_{i=1}^{5} \tau_i\right)^n - \left(\sum_{i=0}^{4} \tau_i\right)^n\right] P_5 + \left[\left(\sum_{i=1}^{6} \tau_i\right)^n - \left(\sum_{i=0}^{5} \tau_i\right)^n\right] P_6 \\ + \left[\left(\sum_{i=1}^{7} \tau_i\right)^n - \left(\sum_{i=0}^{6} \tau_i\right)^n\right] P_7 + \left[\left(\sum_{i=1}^{8} \tau_i\right)^n - \left(\sum_{i=0}^{7} \tau_i\right)^n\right] P_8 + \left[\left(\sum_{i=1}^{9} \tau_i\right)^n - \left(\sum_{i=0}^{8} \tau_i\right)^n\right] P_9 \\ + \left[\left(\sum_{i=1}^{10} \tau_i\right)^n - \left(\sum_{i=0}^{9} \tau_i\right)^n\right] P_{10} + \left[\left(\sum_{i=1}^{11} \tau_i\right)^n - \left(\sum_{i=0}^{10} \tau_i\right)^n\right] P_{11} + \left[\left(\sum_{i=1}^{12} \tau_i\right)^n - \left(\sum_{i=0}^{11} \tau_i\right)^n\right] P_{12} \\ + \left[\left(\sum_{i=1}^{13} \tau_i\right)^n - \left(\sum_{i=0}^{12} \tau_i\right)^n\right] P_{13} + \left[\left(\sum_{i=1}^{14} \tau_i\right)^n - \left(\sum_{i=0}^{13} \tau_i\right)^n\right] P_{14} + \left[\left(\sum_{i=1}^{15} \tau_i\right)^n - \left(\sum_{i=0}^{14} \tau_i\right)^n\right] P_{15} \\ + \left[\left(\sum_{i=1}^{16} \tau_i\right)^n - \left(\sum_{i=0}^{15} \tau_i\right)^n\right] P_{16} + \left[\left(\sum_{i=1}^{17} \tau_i\right)^n - \left(\sum_{i=0}^{16} \tau_i\right)^n\right] P_{17} + \left[\left(\sum_{i=1}^{18} \tau_i\right)^n - \left(\sum_{i=0}^{17} \tau_i\right)^n\right] P_{18} \end{aligned}$$

Theorem:

Let $\tau = \tau_0 + \sum_{i=1}^{18} \tau_i P_i$ be a square 18-plithogenic invertible real matrix, then:

1). $\det(\tau^{-1}) = (\det \tau)^{-1}$

2). $\det \tau^t = \det \tau$

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3). $\det(\tau.Y) = \det\tau.\det Y\,; Y = Y_0 + \sum_{i=1}^{18} Y_i P_i$.

Proof:

1). $det\tau^{-1} = det({\tau_0}^{-1}) + P_1[det(\sum_{i=0}^{1}\tau_i)^{-1} - det({\tau_0}^{-1})] + [det(\sum_{i=0}^{2}\tau_i)^{-1} - det(\sum_{i=0}^{1}\tau_i)^{-1}]P_2 + [det(\sum_{i=1}^{3}\tau_i)^{-1} - det(\sum_{i=0}^{2}\tau_i)^{-1}]P_3 + [det(\sum_{i=1}^{4}\tau_i)^{-1} - det(\sum_{i=0}^{3}\tau_i)^{-1}]P_4 + \left[det\left(\sum_{i=1}^{5}\tau_i\right)^{-1} - det(\sum_{i=0}^{4}\tau_i)^{-1}\right]P_5 + \left[det(\sum_{i=1}^{6}\tau_i)^{-1} - det\left(\sum_{i=0}^{5}\tau_i\right)^{-1}\right]P_6 + [det(\sum_{i=1}^{7}\tau_i)^{-1} - det(\sum_{i=0}^{6}\tau_i)^{-1}]P_7 + [det(\sum_{i=1}^{8}\tau_i)^{-1} - det(\sum_{i=0}^{7}\tau_i)^{-1}]P_8 + [det(\sum_{i=1}^{9}\tau_i)^{-1} - det(\sum_{i=0}^{8}\tau_i)^{-1}]P_9 + [det(\sum_{i=1}^{10}\tau_i)^{-1} - det(\sum_{i=0}^{9}\tau_i)^{-1}]P_{10} + [det(\sum_{i=1}^{11}\tau_i)^{-1} - det(\sum_{i=0}^{10}\tau_i)^{-1}]P_{11} + [det(\sum_{i=1}^{12}\tau_i)^{-1} - det(\sum_{i=0}^{11}\tau_i)^{-1}]P_{12} + [det(\sum_{i=1}^{13}\tau_i)^{-1} - det(\sum_{i=0}^{12}\tau_i)^{-1}]P_{13} + [det(\sum_{i=1}^{14}\tau_i)^{-1} - det(\sum_{i=0}^{13}\tau_i)^{-1}]P_{14} + \left[det\left(\sum_{i=1}^{15}\tau_i\right)^{-1} - det(\sum_{i=0}^{14}\tau_i)^{-1}\right]P_{15} + \left[det(\sum_{i=1}^{16}\tau_i)^{-1} - det\left(\sum_{i=0}^{15}\tau_i\right)^{-1}\right]P_{16} + [det(\sum_{i=1}^{17}\tau_i)^{-1} - det(\sum_{i=0}^{16}\tau_i)^{-1}]P_{17} + [det(\sum_{i=1}^{18}\tau_i)^{-1} - det(\sum_{i=0}^{17}\tau_i)^{-1}]P_{18} = (det\tau)^{-1}$.

2). $\tau^t = {\tau_0}^t + {\tau_1}^t P_1 + {\tau_2}^t P_2 + {\tau_3}^t P_3 + {\tau_4}^t P_4 + {\tau_5}^t P_5 + {\tau_6}^t P_6 + {\tau_7}^t P_7 + {\tau_8}^t P_8 + {\tau_9}^t P_9 + {\tau_{10}}^t P_{10} + {\tau_{11}}^t P_{11} + {\tau_{12}}^t P_{12} + {\tau_{13}}^t P_{13} + {\tau_{14}}^t P_{14} + {\tau_{15}}^t P_{15} + {\tau_{16}}^t P_{16} + {\tau_{17}}^t P_{17} + {\tau_{18}}^t P_{18}$.

$det\tau^t = det({\tau_0}^t) + [det(\sum_{i=0}^{1}{\tau_i}^t) - det({\tau_0}^t)]P_1 + [det(\sum_{i=0}^{2}{\tau_i}^t) - det(\sum_{i=0}^{1}{\tau_i}^t)]P_2 + [det(\sum_{i=0}^{3}{\tau_i}^t) - det(\sum_{i=0}^{2}{\tau_i}^t)]P_3 + [det(\sum_{i=0}^{4}{\tau_i}^t) - det(\sum_{i=0}^{3}{\tau_i}^t)]P_4 + \left[det\left(\sum_{i=0}^{5}{\tau_i}^t\right) - det\ (\sum_{i=0}^{4}{\tau_i}^t)\right]P_5 + \left[det(\sum_{i=0}^{6}{\tau_i}^t) - det\ \left(\sum_{i=0}^{5}{\tau_i}^t\right)\right]P_6 + [det(\sum_{i=0}^{7}{\tau_i}^t) - det\ (\sum_{i=0}^{6}{\tau_i}^t)]P_7 + [det(\sum_{i=0}^{8}{\tau_i}^t) - det\ (\sum_{i=0}^{7}{\tau_i}^t)]P_8 + [det(\sum_{i=0}^{9}{\tau_i}^t) - det\ (\sum_{i=0}^{8}{\tau_i}^t)]P_9 + [det(\sum_{i=0}^{10}{\tau_i}^t) - det\ (\sum_{i=0}^{9}{\tau_i}^t)]P_{10} + [det(\sum_{i=0}^{11}{\tau_i}^t) - det\ (\sum_{i=0}^{10}{\tau_i}^t)]P_{11} + [det(\sum_{i=0}^{12}{\tau_i}^t) - det\ (\sum_{i=0}^{11}{\tau_i}^t)]P_{12} + [det(\sum_{i=0}^{13}{\tau_i}^t) - det\ (\sum_{i=0}^{12}{\tau_i}^t)]P_{13} + [det(\sum_{i=0}^{14}{\tau_i}^t) - det\ (\sum_{i=0}^{13}{\tau_i}^t)]P_{14} + \left[det\left(\sum_{i=0}^{15}{\tau_i}^t\right) - det\ (\sum_{i=0}^{14}{\tau_i}^t)\right]P_{15} + \left[det(\sum_{i=0}^{16}{\tau_i}^t) - det\ \left(\sum_{i=0}^{15}{\tau_i}^t\right)\right]P_{16} + [det(\sum_{i=0}^{17}{\tau_i}^t) - det\ (\sum_{i=0}^{16}{\tau_i}^t)]P_{17} + [det(\sum_{i=0}^{18}{\tau_i}^t) - det\ (\sum_{i=0}^{17}{\tau_i}^t)]P_{18} = \det(\tau_0) + [\det(\sum_{i=0}^{1}\tau_i) - \det(\tau_0)]P_1 + [\det(\sum_{i=0}^{2}\tau_i) - \det(\sum_{i=0}^{1}\tau_i)]P_2 + [\det(\sum_{i=0}^{3}\tau_i) - \det(\sum_{i=0}^{2}\tau_i)]P_3 + [\det(\sum_{i=0}^{4}\tau_i) - \det(\sum_{i=0}^{3}\tau_i)]P_4 + \left[\det\left(\sum_{i=0}^{5}\tau_i\right) - \det(\sum_{i=0}^{4}\tau_i)\right]P_5 + \left[\det(\sum_{i=0}^{6}\tau_i) - \det\left(\sum_{i=0}^{5}\tau_i\right)\right]P_6 + [\det(\sum_{i=0}^{7}\tau_i) - \det(\sum_{i=0}^{6}\tau_i)]P_7 + [\det(\sum_{i=0}^{8}\tau_i) - \det(\sum_{i=0}^{7}\tau_i)]P_8 + [\det(\sum_{i=0}^{9}\tau_i) - \det(\sum_{i=0}^{8}\tau_i)]P_9 + [\det(\sum_{i=0}^{10}\tau_i) - \det(\sum_{i=0}^{9}\tau_i)]P_{10} + [\det(\sum_{i=0}^{11}\tau_i) - \det(\sum_{i=0}^{10}\tau_i)]P_{11} + [\det(\sum_{i=0}^{12}\tau_i) - \det(\sum_{i=0}^{11}\tau_i)]P_{12} + [\det(\sum_{i=0}^{13}\tau_i) - \det(\sum_{i=0}^{12}\tau_i)]P_{13} + [\det(\sum_{i=0}^{14}\tau_i) - \det(\sum_{i=0}^{13}\tau_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15}\tau_i\right) - \det(\sum_{i=0}^{14}\tau_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16}\tau_i) - \det\left(\sum_{i=0}^{15}\tau_i\right)\right]P_{16} + [\det(\sum_{i=0}^{17}\tau_i) - \det(\sum_{i=0}^{16}\tau_i)]P_{17} + [\det(\sum_{i=0}^{18}\tau_i) - \det(\sum_{i=0}^{17}\tau_i)]P_{18} = det\tau$.

3). we have:

$\tau.Y = \tau_0 Y_0 + [\sum_{i=0}^{1}\tau_i \sum_{i=0}^{1} Y_i - \tau_0 Y_0]P_1 + [\sum_{i=0}^{2}\tau_i \sum_{i=0}^{2} Y_i - \sum_{i=0}^{1}\tau_i \sum_{i=0}^{1} Y_i]P_2 + [\sum_{i=0}^{3}\tau_i \sum_{i=0}^{3} Y_i - \sum_{i=0}^{2}\tau_i \sum_{i=0}^{2} Y_i]P_3 + [\sum_{i=0}^{4}\tau_i \sum_{i=0}^{4} Y_i - \sum_{i=0}^{3}\tau_i \sum_{i=0}^{3} Y_i]P_4 + \left[\sum_{i=0}^{5}\tau_i \sum_{i=0}^{5} Y_i - \sum_{i=0}^{4}\tau_i \sum_{i=0}^{4} Y_i\right]P_5 + \left[\sum_{i=0}^{6}\tau_i \sum_{i=0}^{6} Y_i - \sum_{i=0}^{5}\tau_i \sum_{i=0}^{5} Y_i\right]P_6 + [\sum_{i=0}^{7}\tau_i \sum_{i=0}^{7} Y_i - \sum_{i=0}^{6}\tau_i \sum_{i=0}^{6} Y_i]P_7 + [\sum_{i=0}^{8}\tau_i \sum_{i=0}^{8} Y_i - \sum_{i=0}^{7}\tau_i \sum_{i=0}^{7} Y_i]P_8 + [\sum_{i=0}^{9}\tau_i \sum_{i=0}^{9} Y_i - \sum_{i=0}^{8}\tau_i \sum_{i=0}^{8} Y_i]P_9 + [\sum_{i=0}^{10}\tau_i \sum_{i=0}^{10} Y_i - \sum_{i=0}^{9}\tau_i \sum_{i=0}^{9} Y_i]P_{10} + [\sum_{i=0}^{11}\tau_i \sum_{i=0}^{11} Y_i - \sum_{i=0}^{10}\tau_i \sum_{i=0}^{10} Y_i]P_{11} + [\sum_{i=0}^{12}\tau_i \sum_{i=0}^{12} Y_i - \sum_{i=0}^{11}\tau_i \sum_{i=0}^{11} Y_i]P_{12} + [\sum_{i=0}^{13}\tau_i \sum_{i=0}^{13} Y_i - \sum_{i=0}^{12}\tau_i \sum_{i=0}^{12} Y_i]P_{13} + [\sum_{i=0}^{14}\tau_i \sum_{i=0}^{14} Y_i - \sum_{i=0}^{13}\tau_i \sum_{i=0}^{13} Y_i]P_{14} + \left[\sum_{i=0}^{15}\tau_i \sum_{i=0}^{15} Y_i - \sum_{i=0}^{14}\tau_i \sum_{i=0}^{14} Y_i\right]P_{15} + \left[\sum_{i=0}^{16}\tau_i \sum_{i=0}^{16} Y_i - \sum_{i=0}^{15}\tau_i \sum_{i=0}^{15} Y_i\right]P_{16} + [\sum_{i=0}^{17}\tau_i \sum_{i=0}^{17} Y_i - \sum_{i=0}^{16}\tau_i \sum_{i=0}^{16} Y_i]P_{17} + [\sum_{i=0}^{18}\tau_i \sum_{i=0}^{18} Y_i - \sum_{i=0}^{17}\tau_i \sum_{i=0}^{17} Y_i]P_{18}$.

$det(\tau.Y) = det(\tau_0 Y_0) + [det(\sum_{i=0}^{1}\Delta_i \sum_{i=0}^{1} Y_i) - det(\Delta_0 Y_0)]P_1 + [det(\sum_{i=0}^{2}\Delta_i \sum_{i=0}^{2} Y_i) - det(\sum_{i=0}^{1}\tau_i \sum_{i=0}^{1} Y_i)]P_2 + [det(\sum_{i=0}^{3}\tau_i \sum_{i=0}^{3} Y_i) - det(\sum_{i=0}^{2}\tau_i \sum_{i=0}^{2} Y_i)]P_3 + [det(\sum_{i=0}^{4}\tau_i \sum_{i=0}^{4} Y_i) - det(\sum_{i=0}^{3}\tau_i \sum_{i=0}^{3} Y_i)]P_4 + \left[det\left(\sum_{i=0}^{5}\tau_i \sum_{i=0}^{5} Y_i\right) - det\ (\sum_{i=0}^{4}\tau_i \sum_{i=0}^{4} Y_i)\right]P_5 + \left[det(\sum_{i=0}^{6}\tau_i \sum_{i=0}^{6} Y_i) - det\ \left(\sum_{i=0}^{5}\tau_i \sum_{i=0}^{5} Y_i\right)\right]P_6 + [det(\sum_{i=0}^{7}\tau_i \sum_{i=0}^{7} Y_i) - det\ (\sum_{i=0}^{6}\tau_i \sum_{i=0}^{6} Y_i)]P_7 + [det(\sum_{i=0}^{8}\tau_i \sum_{i=0}^{8} Y_i) - det\ (\sum_{i=0}^{7}\tau_i \sum_{i=0}^{7} Y_i)]P_8 + [det(\sum_{i=0}^{9}\tau_i \sum_{i=0}^{9} Y_i) - det\ (\sum_{i=0}^{8}\tau_i \sum_{i=0}^{8} Y_i)]P_9 + [det(\sum_{i=0}^{10}\tau_i \sum_{i=0}^{10} Y_i) - det\ (\sum_{i=0}^{9}\tau_i \sum_{i=0}^{9} Y_i)]P_{10} + [det(\sum_{i=0}^{11}\tau_i \sum_{i=0}^{11} Y_i) - det\ (\sum_{i=0}^{10}\tau_i \sum_{i=0}^{10} Y_i)]P_{11} + [det(\sum_{i=0}^{12}\tau_i \sum_{i=0}^{12} Y_i) - det\ (\sum_{i=0}^{11}\tau_i \sum_{i=0}^{11} Y_i)]P_{12} + [det(\sum_{i=0}^{13}\tau_i \sum_{i=0}^{13} Y_i) - det\ (\sum_{i=0}^{12}\tau_i \sum_{i=0}^{12} Y_i)]P_{13} + [det(\sum_{i=0}^{14}\tau_i \sum_{i=0}^{14} Y_i) - det\ (\sum_{i=0}^{13}\tau_i \sum_{i=0}^{13} Y_i)]P_{14} + \left[det\left(\sum_{i=0}^{15}\tau_i \sum_{i=0}^{15} Y_i\right) - det\ (\sum_{i=0}^{14}\tau_i \sum_{i=0}^{14} Y_i)\right]P_{15} + \left[det(\sum_{i=0}^{16}\tau_i \sum_{i=0}^{16} Y_i) - det\ \left(\sum_{i=0}^{15}\tau_i \sum_{i=0}^{15} C_i\right)\right]P_{16} + [det(\sum_{i=0}^{17}\tau_i \sum_{i=0}^{17} Y_i) - det\ (\sum_{i=0}^{16}\tau_i \sum_{i=0}^{16} C_i)]P_{17} + [det(\sum_{i=0}^{18}\tau_i \sum_{i=0}^{18} Y_i) - det\ (\sum_{i=0}^{17}\tau_i \sum_{i=0}^{17} C_i)]P_{17} = det(\tau_0)det(C_0) + \left[det\left(\sum_{i=0}^{j}\tau_i\right).det\left(\sum_{i=0}^{j} Y_i\right) - det(\tau).det\left(\sum_{i=1}^{j-1} Y_{i-1}\right)\right]P_i = det(\tau)det(Y); 1 \le j \le 18$.

Definition.

Let $\tau = \tau_0 + \sum_{i=1}^{18}\tau_i P_i$ be a symbolic 18-plithogenic real square matrix, then:
τ is called orthogonal if and only if $\tau^t = \tau^{-1}$.

Theorem:

τ is orthogonal if and only if $\sum_{i=0}^{j}\tau_i\,;\ 0 \le j \le 18$ are orthogonal.

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Proof

$\tau^t = \tau^{-1}$, hence:

${\tau_0}^t + \sum_{i=1}^{18} {\tau_i}^t P_i = {\tau_0}^{-1} + [(\sum_{i=0}^{1} \tau_i)^{-1} - {\tau_0}^{-1}]P_1 + [(\sum_{i=0}^{2} \tau_i)^{-1} - (\sum_{i=0}^{1} \tau_i)^{-1}]P_2 + [(\sum_{i=0}^{3} \tau_i)^{-1} - (\sum_{i=0}^{2} \tau_i)^{-1}]P_3 + [(\sum_{i=0}^{4} \tau_i)^{-1} - (\sum_{i=0}^{3} \tau_i)^{-1}]P_4 + \left[\left(\sum_{i=0}^{5} \tau_i\right)^{-1} - (\sum_{i=0}^{4} \tau_i)^{-1}\right] P_5 + \left[(\sum_{i=0}^{6} \tau_i)^{-1} - \left(\sum_{i=0}^{5} \tau_i\right)^{-1}\right] P_6 + [(\sum_{i=0}^{7} \tau_i)^{-1} - (\sum_{i=0}^{6} \tau_i)^{-1}]P_7 + [(\sum_{i=0}^{8} \tau_i)^{-1} - (\sum_{i=0}^{7} \tau_i)^{-1}]P_8 + [(\sum_{i=0}^{9} \tau_i)^{-1} - (\sum_{i=0}^{8} \tau_i)^{-1}]P_9 + [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^{9} \tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + \left[\left(\sum_{i=0}^{15} \tau_i\right)^{-1} - (\sum_{i=0}^{14} \tau_i)^{-1}\right] P_{15} + \left[(\sum_{i=0}^{16} \tau_i)^{-1} - \left(\sum_{i=0}^{15} \tau_i\right)^{-1}\right] P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - (\sum_{i=0}^{17} \tau_i)^{-1}]P_{18}$, thus:

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$$
\begin{cases}
{\tau_0}^{t} = {\tau_0}^{-1} \\
{\tau_1}^{t} = \left(\sum_{i=0}^{1} \tau_i\right)^{-1} - {\tau_0}^{-1} \\
{\tau_2}^{t} = \left(\sum_{i=0}^{2} \tau_i\right)^{-1} - \left(\sum_{i=0}^{1} \tau_i\right)^{-1} \\
{\tau_3}^{t} = \left(\sum_{i=0}^{3} \tau_i\right)^{-1} - \left(\sum_{i=0}^{2} \tau_i\right)^{-1} \\
{\tau_4}^{t} = \left(\sum_{i=0}^{4} \tau_i\right)^{-1} - \left(\sum_{i=0}^{3} \tau_i\right)^{-1} \\
{\tau_5}^{t} = \left(\sum_{i=0}^{5} \tau_i\right)^{-1} - \left(\sum_{i=0}^{4} \tau_i\right)^{-1} \\
{\tau_6}^{t} = \left(\sum_{i=0}^{6} \tau_i\right)^{-1} - \left(\sum_{i=0}^{5} \tau_i\right)^{-1} \\
{\tau_7}^{t} = \left(\sum_{i=0}^{7} \tau_i\right)^{-1} - \left(\sum_{i=0}^{6} \tau_i\right)^{-1} \\
{\tau_8}^{t} = \left(\sum_{i=0}^{8} \tau_i\right)^{-1} - \left(\sum_{i=0}^{7} \tau_i\right)^{-1} \\
{\tau_9}^{t} = \left(\sum_{i=0}^{9} \tau_i\right)^{-1} - \left(\sum_{i=0}^{8} \tau_i\right)^{-1} \\
{\tau_{10}}^{t} = \left(\sum_{i=0}^{10} \tau_i\right)^{-1} - \left(\sum_{i=0}^{9} \tau_i\right)^{-1} \\
{\tau_{11}}^{t} = \left(\sum_{i=0}^{11} \tau_i\right)^{-1} - \left(\sum_{i=0}^{10} \tau_i\right)^{-1} \\
{\tau_{12}}^{t} = \left[\left(\sum_{i=0}^{12} \tau_i\right)^{-1} - \left(\sum_{i=0}^{11} \tau_i\right)^{-1}\right] \\
{\tau_{13}}^{t} = \left(\sum_{i=0}^{13} \tau_i\right)^{-1} - \left(\sum_{i=0}^{12} \tau_i\right)^{-1} \\
{\tau_{14}}^{t} = \left(\sum_{i=0}^{14} \tau_i\right)^{-1} - \left(\sum_{i=0}^{13} \tau_i\right)^{-1} \\
{\tau_{15}}^{t} = \left(\sum_{i=0}^{15} \tau_i\right)^{-1} - \left(\sum_{i=0}^{14} \tau_i\right)^{-1} \\
{\tau_{16}}^{t} = \left(\sum_{i=0}^{16} \tau_i\right)^{-1} - \left(\sum_{i=0}^{15} \tau_i\right)^{-1} \\
{\tau_{17}}^{t} = \left(\sum_{i=0}^{17} \tau_i\right)^{-1} - \left(\sum_{i=0}^{16} \tau_i\right)^{-1} \\
{\tau_{18}}^{t} = \left(\sum_{i=0}^{18} \tau_i\right)^{-1} - \left(\sum_{i=0}^{17} \tau_i\right)^{-1}
\end{cases}
$$

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This implies that:

$$\begin{cases} \boldsymbol{\tau_0}^{t} = \boldsymbol{\tau_0}^{-1} \\ \sum_{i=0}^{1}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{1}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{2}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{2}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{3}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{3}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{4}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{4}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{5}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{5}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{6}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{6}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{7}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{7}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{8}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{8}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{9}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{9}\boldsymbol{\tau_i}\right)^{-1}, \\ \sum_{i=0}^{10}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{10}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{11}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{11}\boldsymbol{\tau}\right)^{-1} \\ \sum_{i=0}^{12}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{12}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{13}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{13}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{14}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{14}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{15}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{15}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{16}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{16}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{17}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{17}\boldsymbol{\tau_i}\right)^{-1} \\ \sum_{i=0}^{18}\boldsymbol{\tau_i}^{t} = \left(\sum_{i=0}^{18}\boldsymbol{\tau_i}\right)^{-1} \end{cases}$$

Definition:

The square 19-plithogenic matrix is defined:

$\tau = \tau_0 + \sum_{i=1}^{19}\tau_i P_i\,; (\tau_i)_{n\times n}$ is square matrix of real entries.

Example.

Consider the symbolic 19-plithogenic matrix:

$$\Delta = \begin{pmatrix} -6 & -9 \\ 1 & 56 \end{pmatrix} + \begin{pmatrix} 1 & 12 \\ 65 & 31 \end{pmatrix} P_1 + \begin{pmatrix} 123 & -1 \\ 12 & 31 \end{pmatrix} P_2 + \begin{pmatrix} -96 & -8 \\ -19 & -16 \end{pmatrix} P_3 + \begin{pmatrix} 38 & 65 \\ 116 & 31 \end{pmatrix} P_4 + \begin{pmatrix} -5 & -5 \\ -5 & -2 \end{pmatrix} P_5 + \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix} P_6 + \begin{pmatrix} -1 & 7 \\ 9 & 8 \end{pmatrix} P_7 + \begin{pmatrix} 12 & 11 \\ 65 & -1 \end{pmatrix} P_8 + \begin{pmatrix} -1 & 19 \\ -1 & 0 \end{pmatrix} P_{10} + \begin{pmatrix} 8 & -1 \\ 7 & 5 \end{pmatrix} P_{11} + \begin{pmatrix} 42 & -41 \\ 2 & -8 \end{pmatrix} P_{12} + \begin{pmatrix} 41 & -1 \\ 21 & -9 \end{pmatrix} P_{13} + \begin{pmatrix} 4 & -11 \\ 21 & -8 \end{pmatrix} P_{14} + \begin{pmatrix} 41 & -1 \\ 2 & -8 \end{pmatrix} P_{15} + \begin{pmatrix} 41 & -31 \\ 32 & -8 \end{pmatrix} P_{16} + \begin{pmatrix} 41 & -1 \\ 2 & -8 \end{pmatrix} P_{17} + \begin{pmatrix} -1 & 19 \\ -1 & 0 \end{pmatrix} P_{18} + \begin{pmatrix} 42 & -41 \\ 2 & -8 \end{pmatrix} P_{19}.$$

Definition.

Let $\tau = \tau_0 + \sum_{i=1}^{19}\tau_i P_i$ be a symbolic 19-plithogenic matrix of size $n \times n$, hence:

$$\det\tau = \det(\tau_0) + [\det(\sum_{i=0}^{1}\tau_i) - \det(\tau_0)]P_1 + [\det(\sum_{i=0}^{2}\tau_i) - \det(\sum_{i=0}^{1}\tau_i)]P_2 + [\det(\sum_{i=0}^{3}\tau_i) - \det(\sum_{i=0}^{2}\tau_i)]P_3 + [\det(\sum_{i=0}^{4}\tau_i) - \det(\sum_{i=0}^{3}\tau_i)]P_4 + \left[\det\left(\sum_{i=0}^{5}\tau_i\right) - \det(\sum_{i=0}^{4}\tau_i)\right]P_5 + \left[\det(\sum_{i=0}^{6}\tau_i) - \det\left(\sum_{i=0}^{5}\tau_i\right)\right]P_6 + [\det(\sum_{i=0}^{7}\tau_i) - \det(\sum_{i=0}^{6}\tau_i)]P_7 + [\det(\sum_{i=0}^{8}\tau_i) - \det(\sum_{i=0}^{7}\tau_i)]P_8 + [\det(\sum_{i=0}^{9}\tau_i) - \det(\sum_{i=0}^{8}\tau_i)]P_9 + [\det(\sum_{i=0}^{10}\tau_i) - \det(\sum_{i=0}^{9}\tau_i)]P_{10} + [\det(\sum_{i=0}^{11}\tau_i) - \det(\sum_{i=0}^{10}\tau_i)]P_{11} + [\det(\sum_{i=0}^{12}\tau_i) - \det(\sum_{i=0}^{11}\tau_i)]P_{12} + [\det(\sum_{i=0}^{13}\tau_i) - \det(\sum_{i=0}^{12}\tau_i)]P_{13} + [\det(\sum_{i=0}^{14}\tau_i) - \det(\sum_{i=0}^{13}\tau_i)]P_{14} + \left[\det\left(\sum_{i=0}^{15}\tau_i\right) - \det(\sum_{i=0}^{14}\tau_i)\right]P_{15} + \left[\det(\sum_{i=0}^{16}\tau_i) - \det\left(\sum_{i=0}^{15}\tau_i\right)\right]P_{16} + [\det(\sum_{i=0}^{17}\tau_i) - \det(\sum_{i=0}^{16}\tau_i)]P_{17} + [\det(\sum_{i=0}^{18}\tau_i) - \det(\sum_{i=0}^{17}\tau_i)]P_{18} + [\det(\sum_{i=0}^{19}\tau_i) - \det(\sum_{i=0}^{18}\tau_i)]P_{19}.$$

Theorem

Let $\tau = \tau_0 + \sum_{i=1}^{19}\tau_i P_i$ be a symbolic 19-plithogenic matrix of size $n \times n$, hence:

1. τ is invertible if and only if $\det\tau$ is an invertible symbolic 19-plithogenic real number.
2. $\tau^{-1} = {\tau_0}^{-1} + [(\sum_{i=0}^{1}\tau_i)^{-1} - {\tau_0}^{-1}]P_1 + [(\sum_{i=0}^{2}\tau_i)^{-1} - (\sum_{i=0}^{1}\tau_i)^{-1}]P_2 + [(\sum_{i=1}^{3}\tau_i)^{-1} - (\sum_{i=0}^{2}\tau_i)^{-1}]P_3 + [(\sum_{i=1}^{4}\tau_i)^{-1} - (\sum_{i=0}^{3}\tau_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5}\tau_i\right)^{-1} - (\sum_{i=0}^{4}\tau_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6}\tau_i)^{-1} - \left(\sum_{i=0}^{5}\tau_i\right)^{-1}\right]P_6 + [(\sum_{i=1}^{7}\tau_i)^{-1} - (\sum_{i=0}^{6}\tau_i)^{-1}]P_7 + [(\sum_{i=1}^{8}\tau_i)^{-1} - (\sum_{i=0}^{7}\tau_i)^{-1}]P_8 + [(\sum_{i=1}^{9}\tau_i)^{-1} -$

$$(\sum_{i=0}^{8}\tau_i)^{-1}]P_9 + [(\sum_{i=1}^{10}\tau_i)^{-1} - (\sum_{i=0}^{9}\tau_i)^{-1}]P_{10} + [(\sum_{i=1}^{11}\tau_i)^{-1} - (\sum_{i=0}^{10}\tau_i)^{-1}]P_{11} + [(\sum_{i=1}^{12}\tau_i)^{-1} -$$
$$(\sum_{i=0}^{11}\tau_i)^{-1}]P_{12} + [(\sum_{i=1}^{13}\tau_i)^{-1} - (\sum_{i=0}^{12}\tau_i)^{-1}]P_{13} + [(\sum_{i=1}^{14}\tau_i)^{-1} - (\sum_{i=0}^{13}\tau_i)^{-1}]P_{14} + \Big[\Big(\sum_{i=1}^{15}\tau_i\Big)^{-1} -$$
$$(\sum_{i=0}^{14}\tau_i)^{-1}\Big] P_{15} + \Big[(\sum_{i=1}^{16}\tau_i)^{-1} - \Big(\sum_{i=0}^{15}\tau_i\Big)^{-1}\Big] P_{16} + [(\sum_{i=1}^{17}\tau_i)^{-1} - (\sum_{i=0}^{16}\tau_i)^{-1}]P_{17} + [(\sum_{i=1}^{18}\tau_i)^{-1} -$$
$$(\sum_{i=0}^{17}\tau_i)^{-1}]P_{18} + [(\sum_{i=1}^{19}\tau_i)^{-1} - (\sum_{i=0}^{18}\tau_i)^{-1}]P_{19}.$$

Proof

1). Let $\tau = \tau_0 + \sum_{i=1}^{19}\tau_i P_i$, then τ is invertible if and only if there exists $I = I_0 + \sum_{i=1}^{19} I_i P_i$ such that:
$\tau \times I = U_{n\times n}$, hence:

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$$\begin{cases}
\tau_0 I_0 = U_{n\times n} \\
\sum_{i=0}^{1}\tau_i\sum_{i=0}^{1}I_i - \tau_0 I_0 = O_{n\times n} \\
\sum_{i=0}^{2}\tau_i\sum_{i=0}^{2}I_i - \sum_{i=0}^{1}\tau_i\sum_{i=0}^{1}I_i = O_{n\times n} \\
\sum_{i=0}^{3}\tau_i\sum_{i=0}^{3}I_i - \sum_{i=0}^{2}\tau_i\sum_{i=0}^{2}I_i = O_{n\times n} \\
\sum_{i=0}^{4}\tau_i\sum_{i=0}^{4}I_i - \sum_{i=0}^{3}\tau_i\sum_{i=0}^{3}I_i = O_{n\times n} \\
\sum_{i=0}^{5}\tau_i\sum_{i=0}^{5}I_i - \sum_{i=0}^{4}\tau_i\sum_{i=0}^{4}I_i = O_{n\times n} \\
\sum_{i=0}^{6}\tau_i\sum_{i=0}^{6}I_i - \sum_{i=0}^{5}\tau_i\sum_{i=0}^{5}I_i = O_{n\times n} \\
\sum_{i=0}^{7}\tau_i\sum_{i=0}^{7}I_i - \sum_{i=0}^{6}\tau_i\sum_{i=0}^{6}I_i = O_{n\times n} \\
\sum_{i=0}^{8}\tau_i\sum_{i=0}^{8}I_i - \sum_{i=0}^{7}\tau_i\sum_{i=0}^{7}I_i = O_{n\times n} \\
\sum_{i=0}^{9}\tau_i\sum_{i=0}^{9}I_i - \sum_{i=0}^{8}\tau_i\sum_{i=0}^{8}I_i = O_{n\times n} \\
\sum_{i=0}^{10}\tau_i\sum_{i=0}^{10}I_i - \sum_{i=0}^{9}\tau_i\sum_{i=0}^{9}I_i = O_{n\times n} \\
\sum_{i=0}^{11}\tau_i\sum_{i=0}^{11}I_i - \sum_{i=0}^{10}\tau_i\sum_{i=0}^{10}I_i = O_{n\times n} \\
\sum_{i=0}^{12}\tau_i\sum_{i=0}^{12}I_i - \sum_{i=0}^{11}\tau_i\sum_{i=0}^{11}I_i = O_{n\times n} \\
\sum_{i=0}^{13}\tau_i\sum_{i=0}^{13}I_i - \sum_{i=0}^{12}\tau_i\sum_{i=0}^{12}I_i = O_{n\times n} \\
\sum_{i=0}^{14}\tau_i\sum_{i=0}^{14}I_i - \sum_{i=0}^{13}\tau_i\sum_{i=0}^{13}I_i = O_{n\times n} \\
\sum_{i=0}^{15}\tau_i\sum_{i=0}^{15}I_i - \sum_{i=0}^{14}\tau_i\sum_{i=0}^{14}I_i = O_{n\times n} \\
\sum_{i=0}^{16}\tau_i\sum_{i=0}^{16}I_i - \sum_{i=0}^{15}\tau_i\sum_{i=0}^{15}I_i = O_{n\times n} \\
\sum_{i=0}^{17}\tau_i\sum_{i=0}^{17}I_i - \sum_{i=0}^{16}\tau_i\sum_{i=0}^{16}I_i = O_{n\times n} \\
\sum_{i=0}^{18}\tau_i\sum_{i=0}^{18}I_i - \sum_{i=0}^{17}\tau_i\sum_{i=0}^{17}I_i = O_{n\times n} \\
\sum_{i=0}^{19}\tau_i\sum_{i=0}^{19}I_i - \sum_{i=0}^{18}\tau_i\sum_{i=0}^{18}I_i = O_{n\times n}
\end{cases}$$

This implies that:

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$$\begin{cases} \tau_0 I_0 = U_{n\times n} \\ \sum_{i=0}^{r} \tau_i \sum_{i=0}^{r} I_i = U_{n\times n} \end{cases} ;\ 1 \le r \le 19$$

Hence $\det(\sum_{i=0}^{r} \tau_i) \neq 0$ for all $1 \le r \le 19$, so that $\det(\tau)$ is invertible in $19 - SP_R$.

2). $\sum_{i=0}^{r} I_i = (\sum_{i=0}^{r} \tau_i)^{-1}$for $1 \le r \le 19$, therefor

$$\tau^{-1} = {\tau_0}^{-1} + [(\sum_{i=0}^{1} \tau_i)^{-1} - {\tau_0}^{-1}]P_1 + [(\sum_{i=0}^{2} \tau_i)^{-1} - (\sum_{i=0}^{1} \tau_i)^{-1}]P_2 + [(\sum_{i=0}^{3} \tau_i)^{-1} - (\sum_{i=0}^{2} \tau_i)^{-1}]P_3 + [(\sum_{i=0}^{4} \tau_i)^{-1} - (\sum_{i=0}^{3} \tau_i)^{-1}]P_4 + \left[(\sum_{i=0}^{5} \tau_i)^{-1} - (\sum_{i=0}^{4} \tau_i)^{-1}\right]P_5 + \left[(\sum_{i=0}^{6} \tau_i)^{-1} - (\sum_{i=0}^{5} \tau_i)^{-1}\right]P_6 + [(\sum_{i=0}^{7} \tau_i)^{-1} - (\sum_{i=0}^{6} \tau_i)^{-1}]P_7 + [(\sum_{i=0}^{8} \tau_i)^{-1} - (\sum_{i=0}^{7} \tau_i)^{-1}]P_8 + [(\sum_{i=0}^{9} \tau_i)^{-1} - (\sum_{i=0}^{8} \tau_i)^{-1}]P_9 + [(\sum_{i=0}^{10} \tau_i)^{-1} - (\sum_{i=0}^{9} \tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11} \tau_i)^{-1} - (\sum_{i=0}^{10} \tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12} \tau_i)^{-1} - (\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + \left[(\sum_{i=0}^{15} \tau_i)^{-1} - (\sum_{i=0}^{14} \tau_i)^{-1}\right]P_{15} + \left[(\sum_{i=0}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}\right]P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} - (\sum_{i=0}^{17} \tau_i)^{-1}]P_{18} + [(\sum_{i=0}^{19} \tau_i)^{-1} - (\sum_{i=0}^{18} \tau_i)^{-1}]P_{19}.$$

Definition.

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{19} \varepsilon_i P_i$ be a symbolic 19-plithogenic real number and $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a symbolic 19-plithogenic square real matrix, then ε is called symbolic 18-plithogenic eigen value if and only if $\tau X = \varepsilon X$.
X is called symbolic 19-plithogenic eigenvector.

Theorem:

Let $\varepsilon = \varepsilon_0 + \sum_{i=1}^{19} \varepsilon_i P_i \in 19 - SP_R$, $X = X_0 + \sum_{i=1}^{19} X_i P_i$ be a symbolic 19-plithogenic vector, then ε is eigen value of $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ with X as the corresponding eigen vector if and only if:
$\sum_{i=0}^{j} \varepsilon_i$ is eigen value of $\sum_{i=0}^{j} \tau_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \le j \le 19$.

Proof:

It is clear that r is an eigen value of τ with X as an eigen vector if and only if:
$\tau . X = \varepsilon . X$, which is equivalent to:

$$\begin{cases} \tau_0 X_0 = \varepsilon_0 X_0 \\ \sum_{i=0}^{j} \tau_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} \varepsilon_i \sum_{i=0}^{j} X_i \end{cases} ;\ 1 \le j \le 19$$

Which is equivalent to:
$\sum_{i=0}^{j} \varepsilon_i$ is an eigen value of $\sum_{i=0}^{j} \tau_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 19$.

Theorem:

$$\begin{aligned} \tau^n = {\tau_0}^n + P_1\left[\left(\sum_{i=0}^{1} \tau_i\right)^n - {\tau_0}^n\right] + \left[\left(\sum_{i=0}^{2} \tau_i\right)^n - \left(\sum_{i=0}^{1} \tau_i\right)^n\right]P_2 + \left[\left(\sum_{i=0}^{3} \tau_i\right)^n - \left(\sum_{i=0}^{2} \tau_i\right)^n\right]P_3 \\ + \left[\left(\sum_{i=0}^{4} \tau_i\right)^n - \left(\sum_{i=0}^{3} \tau_i\right)^n\right]P_4 + \left[\left(\sum_{i=1}^{5} \tau_i\right)^n - \left(\sum_{i=0}^{4} \tau_i\right)^n\right]P_5 + \left[\left(\sum_{i=1}^{6} \tau_i\right)^n - \left(\sum_{i=0}^{5} \tau_i\right)^n\right]P_6 \\ + \left[\left(\sum_{i=1}^{7} \tau_i\right)^n - \left(\sum_{i=0}^{6} \tau_i\right)^n\right]P_7 + \left[\left(\sum_{i=1}^{8} \tau_i\right)^n - \left(\sum_{i=0}^{7} \tau_i\right)^n\right]P_8 + \left[\left(\sum_{i=1}^{9} \tau_i\right)^n - \left(\sum_{i=0}^{8} \tau_i\right)^n\right]P_9 \\ + \left[\left(\sum_{i=1}^{10} \tau_i\right)^n - \left(\sum_{i=0}^{9} \tau_i\right)^n\right]P_{10} + \left[\left(\sum_{i=1}^{11} \tau_i\right)^n - \left(\sum_{i=0}^{10} \tau_i\right)^n\right]P_{11} + \left[\left(\sum_{i=1}^{12} \tau_i\right)^n - \left(\sum_{i=0}^{11} \tau_i\right)^n\right]P_{12} \\ + \left[\left(\sum_{i=1}^{13} \tau_i\right)^n - \left(\sum_{i=0}^{12} \tau_i\right)^n\right]P_{13} + \left[\left(\sum_{i=1}^{14} \tau_i\right)^n - \left(\sum_{i=0}^{13} \tau_i\right)^n\right]P_{14} + \left[\left(\sum_{i=1}^{15} \tau_i\right)^n - \left(\sum_{i=0}^{14} \tau_i\right)^n\right]P_{15} \\ + \left[\left(\sum_{i=1}^{16} \tau_i\right)^n - \left(\sum_{i=0}^{15} \tau_i\right)^n\right]P_{16} + \left[\left(\sum_{i=1}^{17} \tau_i\right)^n - \left(\sum_{i=0}^{16} \tau_i\right)^n\right]P_{17} + \left[\left(\sum_{i=1}^{18} \tau_i\right)^n - \left(\sum_{i=0}^{17} \tau_i\right)^n\right]P_{18} \\ + \left[\left(\sum_{i=1}^{19} \tau_i\right)^n - \left(\sum_{i=0}^{18} \tau_i\right)^n\right]P_{19} \end{aligned}$$

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Theorem:
Let $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a square 19-plithogenic invertible real matrix, then:
1). $\det(\tau^{-1}) = (\det \tau)^{-1}$
2). $\det \tau^t = \det \tau$
3). $\det(\tau . Y) = \det \tau . \det Y \,; Y = Y_0 + \sum_{i=1}^{19} Y_i P_i$.
Proof:
1). $det\tau^{-1} = det({\tau_0}^{-1}) + P_1[det(\sum_{i=0}^{1}\tau_i)^{-1} - det({\tau_0}^{-1})] + [det(\sum_{i=0}^{2}\tau_i)^{-1} - det(\sum_{i=0}^{1}\tau_i)^{-1}]P_2 +$
$[det(\sum_{i=1}^{3}\tau_i)^{-1} - det(\sum_{i=0}^{2}\tau_i)^{-1}]P_3 + [det(\sum_{i=1}^{4}\tau_i)^{-1} - det(\sum_{i=0}^{3}\tau_i)^{-1}]P_4 + \Big[det\big(\sum_{i=1}^{5}\tau_i\big)^{-1} -$
$det(\sum_{i=0}^{4}\tau_i)^{-1}\Big] P_5 + \Big[det(\sum_{i=1}^{6}\tau_i)^{-1} - det\big(\sum_{i=0}^{5}\tau_i\big)^{-1}\Big] P_6 + [det(\sum_{i=1}^{7}\tau_i)^{-1} - det(\sum_{i=0}^{6}\tau_i)^{-1}]P_7 +$
$[det(\sum_{i=1}^{8}\tau_i)^{-1} - det(\sum_{i=0}^{7}\tau_i)^{-1}]P_8 + [det(\sum_{i=1}^{9}\tau_i)^{-1} - det(\sum_{i=0}^{8}\tau_i)^{-1}]P_9 + [det(\sum_{i=1}^{10}\tau_i)^{-1} -$
$det(\sum_{i=0}^{9}\tau_i)^{-1}]P_{10} + [det(\sum_{i=1}^{11}\tau_i)^{-1} - det(\sum_{i=0}^{10}\tau_i)^{-1}]P_{11} + [det(\sum_{i=1}^{12}\tau_i)^{-1} - det(\sum_{i=0}^{11}\tau_i)^{-1}]P_{12} +$
$[det(\sum_{i=1}^{13}\tau_i)^{-1} - det(\sum_{i=0}^{12}\tau_i)^{-1}]P_{13} + [det(\sum_{i=1}^{14}\tau_i)^{-1} - det(\sum_{i=0}^{13}\tau_i)^{-1}]P_{14} + \Big[det\big(\sum_{i=1}^{15}\tau_i\big)^{-1} -$
$det(\sum_{i=0}^{14}\tau_i)^{-1}\Big] P_{15} + \Big[det(\sum_{i=1}^{16}\tau_i)^{-1} - det\big(\sum_{i=0}^{15}\tau_i\big)^{-1}\Big] P_{16} + [det(\sum_{i=1}^{17}\tau_i)^{-1} - det(\sum_{i=0}^{16}\tau_i)^{-1}]P_{17} +$
$[det(\sum_{i=1}^{18}\tau_i)^{-1} - det(\sum_{i=0}^{17}\tau_i)^{-1}]P_{18} + [det(\sum_{i=1}^{19}\tau_i)^{-1} - det(\sum_{i=0}^{18}\tau_i)^{-1}]P_{19} = (det\tau)^{-1}.$
2). $\tau^t = {\tau_0}^t + {\tau_1}^t P_1 + {\tau_2}^t P_2 + {\tau_3}^t P_3 + {\tau_4}^t P_4 + {\tau_5}^t P_5 + {\tau_6}^t P_6 + {\tau_7}^t P_7 + {\tau_8}^t P_8 + {\tau_9}^t P_9 + {\tau_{10}}^t P_{10} + {\tau_{11}}^t P_{11} +$
${\tau_{12}}^t P_{12} + {\tau_{13}}^t P_{13} + {\tau_{14}}^t P_{14} + {\tau_{15}}^t P_{15} + {\tau_{16}}^t P_{16} + {\tau_{17}}^t P_{17} + {\tau_{18}}^t P_{18} + {\tau_{19}}^t P_{19}.$
$det\tau^t = det({\tau_0}^t) + [det(\sum_{i=0}^{1}{\tau_i}^t) - det({\tau_0}^t)]P_1 + [det(\sum_{i=0}^{2}{\tau_i}^t) - det(\sum_{i=0}^{1}{\tau_i}^t)]P_2 + [det(\sum_{i=0}^{3}{\tau_i}^t) -$
$det(\sum_{i=0}^{2}{\tau_i}^t)]P_3 + [det(\sum_{i=0}^{4}{\tau_i}^t) - det(\sum_{i=0}^{3}{\tau_i}^t)]P_4 + \big[det\big(\sum_{i=0}^{5}{\tau_i}^t\big) - det\ (\sum_{i=0}^{4}{\tau_i}^t)\big]P_5 + \big[det(\sum_{i=0}^{6}{\tau_i}^t) -$
$det\ \big(\sum_{i=0}^{5}{\tau_i}^t\big)\big]P_6 + [det(\sum_{i=0}^{7}{\tau_i}^t) - det\ (\sum_{i=0}^{6}{\tau_i}^t)]P_7 + [det(\sum_{i=0}^{8}{\tau_i}^t) - det\ (\sum_{i=0}^{7}{\tau_i}^t)]P_8 +$
$[det(\sum_{i=0}^{9}{\tau_i}^t) - det\ (\sum_{i=0}^{8}{\tau_i}^t)]P_9 + [det(\sum_{i=0}^{10}{\tau_i}^t) - det\ (\sum_{i=0}^{9}{\tau_i}^t)]P_{10} + [det(\sum_{i=0}^{11}{\tau_i}^t) -$
$det\ (\sum_{i=0}^{10}{\tau_i}^t)]P_{11} + [det(\sum_{i=0}^{12}{\tau_i}^t) - det\ (\sum_{i=0}^{11}{\tau_i}^t)]P_{12} + [det(\sum_{i=0}^{13}{\tau_i}^t) - det\ (\sum_{i=0}^{12}{\tau_i}^t)]P_{13} +$
$[det(\sum_{i=0}^{14}{\tau_i}^t) - det\ (\sum_{i=0}^{13}{\tau_i}^t)]P_{14} + \big[det\big(\sum_{i=0}^{15}{\tau_i}^t\big) - det\ (\sum_{i=0}^{14}{\tau_i}^t)\big]P_{15} + \big[det(\sum_{i=0}^{16}{\tau_i}^t) -$
$det\ \big(\sum_{i=0}^{15}{\tau_i}^t\big)\big]P_{16} + [det(\sum_{i=0}^{17}{\tau_i}^t) - det\ (\sum_{i=0}^{16}{\tau_i}^t)]P_{17} + [det(\sum_{i=0}^{18}{\tau_i}^t) - det\ (\sum_{i=0}^{17}{\tau_i}^t)]P_{18} +$
$[det(\sum_{i=0}^{19}{\tau_i}^t) - det\ (\sum_{i=0}^{18}{\tau_i}^t)]P_{19} = \det(\tau_0) + [\det(\sum_{i=0}^{1}\tau_i) - \det(\tau_0)]P_1 + [\det(\sum_{i=0}^{2}\tau_i) -$
$\det(\sum_{i=0}^{1}\tau_i)]P_2 + [\det(\sum_{i=0}^{3}\tau_i) - \det(\sum_{i=0}^{2}\tau_i)]P_3 + [\det(\sum_{i=0}^{4}\tau_i) - \det(\sum_{i=0}^{3}\tau_i)]P_4 + \big[\det\big(\sum_{i=0}^{5}\tau_i\big) -$
$\det(\sum_{i=0}^{4}\tau_i)\big]P_5 + \big[\det(\sum_{i=0}^{6}\tau_i) - \det\big(\sum_{i=0}^{5}\tau_i\big)\big]P_6 + [\det(\sum_{i=0}^{7}\tau_i) - \det(\sum_{i=0}^{6}\tau_i)]P_7 + [\det(\sum_{i=0}^{8}\tau_i) -$
$\det(\sum_{i=0}^{7}\tau_i)]P_8 + [\det(\sum_{i=0}^{9}\tau_i) - \det(\sum_{i=0}^{8}\tau_i)]P_9 + [\det(\sum_{i=0}^{10}\tau_i) - \det(\sum_{i=0}^{9}\tau_i)]P_{10} + [\det(\sum_{i=0}^{11}\tau_i) -$
$\det(\sum_{i=0}^{10}\tau_i)]P_{11} + [\det(\sum_{i=0}^{12}\tau_i) - \det(\sum_{i=0}^{11}\tau_i)]P_{12} + [\det(\sum_{i=0}^{13}\tau_i) - \det(\sum_{i=0}^{12}\tau_i)]P_{13} + [\det(\sum_{i=0}^{14}\tau_i) -$
$\det(\sum_{i=0}^{13}\tau_i)]P_{14} + \big[\det\big(\sum_{i=0}^{15}\tau_i\big) - \det(\sum_{i=0}^{14}\tau_i)\big]P_{15} + \big[\det(\sum_{i=0}^{16}\tau_i) - \det\big(\sum_{i=0}^{15}\tau_i\big)\big]P_{16} + [\det(\sum_{i=0}^{17}\tau_i) -$
$\det(\sum_{i=0}^{16}\tau_i)]P_{17} + [\det(\sum_{i=0}^{18}\tau_i) - \det(\sum_{i=0}^{17}\tau_i)]P_{18} + [\det(\sum_{i=0}^{19}\tau_i) - \det(\sum_{i=0}^{18}\tau_i)]P_{19} = det\tau.$
3). we have:
$\tau . Y = \tau_0 Y_0 + [\sum_{i=0}^{1}\tau_i \sum_{i=0}^{1} Y_i - \tau_0 Y_0]P_1 + [\sum_{i=0}^{2}\tau_i \sum_{i=0}^{2} Y_i - \sum_{i=0}^{1}\tau_i \sum_{i=0}^{1} Y_i]P_2 + [\sum_{i=0}^{3}\tau_i \sum_{i=0}^{3} Y_i -$
$\sum_{i=0}^{2}\tau_i \sum_{i=0}^{2} Y_i]P_3 + [\sum_{i=0}^{4}\tau_i \sum_{i=0}^{4} Y_i - \sum_{i=0}^{3}\tau_i \sum_{i=0}^{3} Y_i]P_4 + \big[\sum_{i=0}^{5}\tau_i \sum_{i=0}^{5} Y_i - \sum_{i=0}^{4}\tau_i \sum_{i=0}^{4} Y_i\big]P_5 +$
$\big[\sum_{i=0}^{6}\tau_i \sum_{i=0}^{6} Y_i - \sum_{i=0}^{5}\tau_i \sum_{i=0}^{5} Y_i\big]P_6 + [\sum_{i=0}^{7}\tau_i \sum_{i=0}^{7} Y_i - \sum_{i=0}^{6}\tau_i \sum_{i=0}^{6} Y_i]P_7 + [\sum_{i=0}^{8}\tau_i \sum_{i=0}^{8} Y_i -$
$\sum_{i=0}^{7}\tau_i \sum_{i=0}^{7} Y_i]P_8 + [\sum_{i=0}^{9}\tau_i \sum_{i=0}^{9} Y_i - \sum_{i=0}^{8}\tau_i \sum_{i=0}^{8} Y_i]P_9 + [\sum_{i=0}^{10}\tau_i \sum_{i=0}^{10} Y_i - \sum_{i=0}^{9}\tau_i \sum_{i=0}^{9} Y_i]P_{10} +$
$[\sum_{i=0}^{11}\tau_i \sum_{i=0}^{11} Y_i - \sum_{i=0}^{10}\tau_i \sum_{i=0}^{10} Y_i]P_{11} + [\sum_{i=0}^{12}\tau_i \sum_{i=0}^{12} Y_i - \sum_{i=0}^{11}\tau_i \sum_{i=0}^{11} Y_i]P_{12} + [\sum_{i=0}^{13}\tau_i \sum_{i=0}^{13} Y_i -$
$\sum_{i=0}^{12}\tau_i \sum_{i=0}^{12} Y_i]P_{13} + [\sum_{i=0}^{14}\tau_i \sum_{i=0}^{14} Y_i - \sum_{i=0}^{13}\tau_i \sum_{i=0}^{13} Y_i]P_{14} + \big[\sum_{i=0}^{15}\tau_i \sum_{i=0}^{15} Y_i - \sum_{i=0}^{14}\tau_i \sum_{i=0}^{14} Y_i\big]P_{15} +$
$\big[\sum_{i=0}^{16}\tau_i \sum_{i=0}^{16} Y_i - \sum_{i=0}^{15}\tau_i \sum_{i=0}^{15} Y_i\big]P_{16} + [\sum_{i=0}^{17}\tau_i \sum_{i=0}^{17} Y_i - \sum_{i=0}^{16}\tau_i \sum_{i=0}^{16} Y_i]P_{17} + [\sum_{i=0}^{18}\tau_i \sum_{i=0}^{18} Y_i -$
$\sum_{i=0}^{17}\tau_i \sum_{i=0}^{17} Y_i]P_{18} + [\sum_{i=0}^{19}\tau_i \sum_{i=0}^{19} Y_i - \sum_{i=0}^{18}\tau_i \sum_{i=0}^{18} Y_i]P_{19}.$
$det(\tau . Y) = det(\tau_0)det(C_0) + \big[det\big(\sum_{i=0}^{j}\tau_i\big). det\big(\sum_{i=0}^{j} Y_i\big) - det(\tau). det\big(\sum_{i=1}^{j-1} Y_{i-1}\big)\big]P_i = det(\tau)det(Y); 1 \le$
$j \le 19.$

Definition.
Let $\tau = \tau_0 + \sum_{i=1}^{19} \tau_i P_i$ be a symbolic 19-plithogenic real square matrix, then:
τ is called orthogonal if and only if $\tau^t = \tau^{-1}$.
Theorem:
τ is orthogonal if and only if $\sum_{i=0}^{j}\tau_i \,;\ 0 \le j \le 19$ are orthogonal.
Proof
$\tau^t = \tau^{-1}$, hence:
${\tau_0}^t + \sum_{i=1}^{19} {\tau_i}^t P_i = {\tau_0}^{-1} + [(\sum_{i=0}^{1}\tau_i)^{-1} - {\tau_0}^{-1}]P_1 + [(\sum_{i=0}^{2}\tau_i)^{-1} - (\sum_{i=0}^{1}\tau_i)^{-1}]P_2 + [(\sum_{i=0}^{3}\tau_i)^{-1} -$
$(\sum_{i=0}^{2}\tau_i)^{-1}]P_3 + [(\sum_{i=0}^{4}\tau_i)^{-1} - (\sum_{i=0}^{3}\tau_i)^{-1}]P_4 + \Big[\big(\sum_{i=0}^{5}\tau_i\big)^{-1} - (\sum_{i=0}^{4}\tau_i)^{-1}\Big] P_5 + \Big[(\sum_{i=0}^{6}\tau_i)^{-1} -$
$\big(\sum_{i=0}^{5}\tau_i\big)^{-1}\Big] P_6 + [(\sum_{i=0}^{7}\tau_i)^{-1} - (\sum_{i=0}^{6}\tau_i)^{-1}]P_7 + [(\sum_{i=0}^{8}\tau_i)^{-1} - (\sum_{i=0}^{7}\tau_i)^{-1}]P_8 + [(\sum_{i=0}^{9}\tau_i)^{-1} -$
$(\sum_{i=0}^{8}\tau_i)^{-1}]P_9 + [(\sum_{i=0}^{10}\tau_i)^{-1} - (\sum_{i=0}^{9}\tau_i)^{-1}]P_{10} + [(\sum_{i=0}^{11}\tau_i)^{-1} - (\sum_{i=0}^{10}\tau_i)^{-1}]P_{11} + [(\sum_{i=0}^{12}\tau_i)^{-1} -$

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$(\sum_{i=0}^{11} \tau_i)^{-1}]P_{12} + [(\sum_{i=0}^{13} \tau_i)^{-1} - (\sum_{i=0}^{12} \tau_i)^{-1}]P_{13} + [(\sum_{i=0}^{14} \tau_i)^{-1} - (\sum_{i=0}^{13} \tau_i)^{-1}]P_{14} + \left[(\sum_{i=0}^{15} \tau_i)^{-1} -\right.$ $\left.(\sum_{i=0}^{14} \tau_i)^{-1}\right] P_{15} + \left[(\sum_{i=0}^{16} \tau_i)^{-1} - (\sum_{i=0}^{15} \tau_i)^{-1}\right] P_{16} + [(\sum_{i=0}^{17} \tau_i)^{-1} - (\sum_{i=0}^{16} \tau_i)^{-1}]P_{17} + [(\sum_{i=0}^{18} \tau_i)^{-1} -$ $(\sum_{i=0}^{17} \tau_i)^{-1}]P_{18} + [(\sum_{i=0}^{19} \tau_i)^{-1} - (\sum_{i=0}^{18} \tau_i)^{-1}]P_{19}$, thus:

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$$\left\{\begin{array}{l}
{\tau_0}^t = {\tau_0}^{-1} \\
{\tau_1}^t = \left(\sum_{i=0}^{1} \tau_i\right)^{-1} - {\tau_0}^{-1} \\
{\tau_2}^t = \left(\sum_{i=0}^{2} \tau_i\right)^{-1} - \left(\sum_{i=0}^{1} \tau_i\right)^{-1} \\
{\tau_3}^t = \left(\sum_{i=0}^{3} \tau_i\right)^{-1} - \left(\sum_{i=0}^{2} \tau_i\right)^{-1} \\
{\tau_4}^t = \left(\sum_{i=0}^{4} \tau_i\right)^{-1} - \left(\sum_{i=0}^{3} \tau_i\right)^{-1} \\
{\tau_5}^t = \left(\sum_{i=0}^{5} \tau_i\right)^{-1} - \left(\sum_{i=0}^{4} \tau_i\right)^{-1} \\
{\tau_6}^t = \left(\sum_{i=0}^{6} \tau_i\right)^{-1} - \left(\sum_{i=0}^{5} \tau_i\right)^{-1} \\
{\tau_7}^t = \left(\sum_{i=0}^{7} \tau_i\right)^{-1} - \left(\sum_{i=0}^{6} \tau_i\right)^{-1} \\
{\tau_8}^t = \left(\sum_{i=0}^{8} \tau_i\right)^{-1} - \left(\sum_{i=0}^{7} \tau_i\right)^{-1} \\
{\tau_9}^t = \left(\sum_{i=0}^{9} \tau_i\right)^{-1} - \left(\sum_{i=0}^{8} \tau_i\right)^{-1} \\
{\tau_{10}}^t = \left(\sum_{i=0}^{10} \tau_i\right)^{-1} - \left(\sum_{i=0}^{9} \tau_i\right)^{-1} \\
{\tau_{11}}^t = \left(\sum_{i=0}^{11} \tau_i\right)^{-1} - \left(\sum_{i=0}^{10} \tau_i\right)^{-1} \\
{\tau_{12}}^t = \left[\left(\sum_{i=0}^{12} \tau_i\right)^{-1} - \left(\sum_{i=0}^{11} \tau_i\right)^{-1}\right] \\
{\tau_{13}}^t = \left(\sum_{i=0}^{13} \tau_i\right)^{-1} - \left(\sum_{i=0}^{12} \tau_i\right)^{-1} \\
{\tau_{14}}^t = \left(\sum_{i=0}^{14} \tau_i\right)^{-1} - \left(\sum_{i=0}^{13} \tau_i\right)^{-1} \\
{\tau_{15}}^t = \left(\sum_{i=0}^{15} \tau_i\right)^{-1} - \left(\sum_{i=0}^{14} \tau_i\right)^{-1} \\
{\tau_{16}}^t = \left(\sum_{i=0}^{16} \tau_i\right)^{-1} - \left(\sum_{i=0}^{15} \tau_i\right)^{-1} \\
{\tau_{17}}^t = \left(\sum_{i=0}^{17} \tau_i\right)^{-1} - \left(\sum_{i=0}^{16} \tau_i\right)^{-1} \\
{\tau_{18}}^t = \left(\sum_{i=0}^{18} \tau_i\right)^{-1} - \left(\sum_{i=0}^{17} \tau_i\right)^{-1} \\
{\tau_{19}}^t = \left(\sum_{i=0}^{19} \tau_i\right)^{-1} - \left(\sum_{i=0}^{18} \tau_i\right)^{-1}
\end{array}\right.$$

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This implies that:

$$\begin{cases} \tau_0{}^{t} = \tau_0{}^{-1} \\ \sum_{i=0}^{1} \tau_i{}^{t} = \left(\sum_{i=0}^{1} \tau_i\right)^{-1} \\ \sum_{i=0}^{2} \tau_i{}^{t} = \left(\sum_{i=0}^{2} \tau_i\right)^{-1} \\ \sum_{i=0}^{3} \tau_i{}^{t} = \left(\sum_{i=0}^{3} \tau_i\right)^{-1} \\ \sum_{i=0}^{4} \tau_i{}^{t} = \left(\sum_{i=0}^{4} \tau_i\right)^{-1} \\ \sum_{i=0}^{5} \tau_i{}^{t} = \left(\sum_{i=0}^{5} \tau_i\right)^{-1} \\ \sum_{i=0}^{6} \tau_i{}^{t} = \left(\sum_{i=0}^{6} \tau_i\right)^{-1} \\ \sum_{i=0}^{7} \tau_i{}^{t} = \left(\sum_{i=0}^{7} \tau_i\right)^{-1} \\ \sum_{i=0}^{8} \tau_i{}^{t} = \left(\sum_{i=0}^{8} \tau_i\right)^{-1} \\ \sum_{i=0}^{9} \tau_i{}^{t} = \left(\sum_{i=0}^{9} \tau_i\right)^{-1} \\ \sum_{i=0}^{10} \tau_i{}^{t} = \left(\sum_{i=0}^{10} \tau_i\right)^{-1}, \\ \sum_{i=0}^{11} \tau_i{}^{t} = \left(\sum_{i=0}^{11} \tau\right)^{-1} \\ \sum_{i=0}^{12} \tau_i{}^{t} = \left(\sum_{i=0}^{12} \tau_i\right)^{-1} \\ \sum_{i=0}^{13} \tau_i{}^{t} = \left(\sum_{i=0}^{13} \tau_i\right)^{-1} \\ \sum_{i=0}^{14} \tau_i{}^{t} = \left(\sum_{i=0}^{14} \tau_i\right)^{-1} \\ \sum_{i=0}^{15} \tau_i{}^{t} = \left(\sum_{i=0}^{15} \tau_i\right)^{-1} \\ \sum_{i=0}^{16} \tau_i{}^{t} = \left(\sum_{i=0}^{16} \tau_i\right)^{-1} \\ \sum_{i=0}^{17} \tau_i{}^{t} = \left(\sum_{i=0}^{17} \tau_i\right)^{-1} \\ \sum_{i=0}^{18} \tau_i{}^{t} = \left(\sum_{i=0}^{18} \tau_i\right)^{-1} \\ \sum_{i=0}^{19} \tau_i{}^{t} = \left(\sum_{i=0}^{19} \tau_i\right)^{-1} \end{cases}$$

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